Wave equation versus ray based imaging

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At the recent EAGE meeting in Florence, an interactive session entitled 'Wave equation versus ray trace imaging' was set-up by Laurence Nicoletis (Institut Français du Pétrole) and organized by Ian Jones and Gilles Lambaré . In the following article, the co-chairmen review the session, summarizing the salient details.

Introduction

Recently much attention has been given to so called 'wave equation' migration, and its advantage over Kirchhoff techniques. The objective of this interactive session was to contrast and compare various migration schemes to assess the inherent limitations (if any) on each of them: was news of 'the death of Kirchhoff' premature?

By the beginning of the nineties 3D prestack depth imaging became tractable due the appearance of efficient first-arrival travel-time solvers. Further, the ability of Kirchhoff (and other integral schemes) to produce limited subsets of data made industrial application affordable, especially given the iterative nature of model building. Limitations of such Kirchhoff migration codes rapidly became evident for complex media, but at that time 3D wavefield continuation migration remained unaffordable. As a consequence, efforts went towards the improvement of Kirchhoff migration, and some tractable solutions were proposed using multi-arrival travel times and better dealing with amplitudes.

The first item of contention here is the terminology: all migrations are meant to be solutions of the 'wave equation' and so it is imprecise to exclude Kirchhoff schemes from this terminology. Some alternative suggestions were to refer to the alternatives as 'integral' versus 'differential' ('extrapolation' or 'continuation') schemes.

For the bulk of this note, we will refer to the differential techniques as Wavefield Extrapolation (**WE**), as this abbreviation is interchangeable with the commonly used name of Wave Equation migration.

The Talks

The session started with introductory overviews by the chairmen: Ian Jones briefly summarized the advantages and limitations of both approaches (Table 1). Then Gilles Lambaré reviewed the recent evolution of the subject, showing examples of various implementations.

John Etgen (bp Upstream Technology) was the first speaker. His talk was centered around the general question ***"Is wave equation migration ready to replace Kirchhoff migration?"*** John provided several comparisons of images obtained by WE migration and Kirchhoff migration.

The higher quality of images obtained by WE migration could be generally recognized. However strongly dipping reflectors were, with no surprise, better imaged by Kirchhoff migration. John asserted that several aspects of WE migration still had to be improved: amplitude preservation, computation of Common Image Gathers (CIGs), CPU efficiency, and preservation of dipping reflectors.

As second speaker Dan Kosloff (Paradigm Geophysical) gave a general overview of the experience gained by a contractor: ***"Kirchhoff versus wave equation pre stack depth migration"***. (co-authored with Zvi Koren, Alex Litvin, Evgeny Ragoza, & Alexei Zuev). He showed various comparative studies, for which WE migration generally provided a better result. Concerning the capabilities of Kirchhoff migration Dan presented high quality images obtained by 2D multi-arrival preserved amplitude migration on the Marmousi dataset. He noted that there was no reason for such results not to generalize to 3D. In other words, Kirchhoff and WE techniques give comparable results if high fidelity algorithms are used.

Finally the first part of the session was closed by a talk by Biondo Biondi (Stanford University) entitled "***3D pre stack wave equation imaging: a rapidly evolving technology***" (co-authored with Robert G. Clapp, Paul Sava, Marie Prucha). Biondo outlined the recent evolution of WE migration and the general perspectives in terms of amplitude preservation, CIGs and WE migration based velocity macro-model estimation. In addition to the various results demonstrating the quality of images obtained by WE migration, Biondo strongly insisted on the necessary consistency of velocity macro-model estimation and imaging processes: if we migrate using WE then we should model build using the same techniques.

The first discussion period brought together the first three speakers, the two chairmen and the audience of the interactive session. First it was emphasized that having a fair comparison of WE migration and ray-based migration was certainly not easy, since efficiency and quality of both approaches strongly relied on the specificity of the numerical implementations, and on the relevance of the pre-processing sequences. It was particularly true for Kirchhoff migration, for which it would have been particularly interesting to specify in detail the algorithms involved in the comparative studies. Having this point in mind, it would appear questionable to draw general conclusions.

Gilles Lambaré noted that ray based approaches all rely on high frequency approximations. They can be used with some benefit as soon as you can identify events in the data cube. For depth imaging the use of high frequency asymptotic solutions brought many benefits. First of all the reduction of the wave propagation problem to that of the separate computation of travel times and amplitudes, greatly reduced computing time. This was the main reason permitting the cost-effective development of 3D Kirchhoff prestack migration. However the advantages of ray theory can't be reduced to this important point alone. Indeed ray theory facilitates understanding of many aspects of the physics of seismic imaging. Quantitative imaging and common angle imaging have both been introduced using assumptions based on the high frequency approximation.

Gilles also asserted that although high frequency approximations had limitations, most of the artefacts observed in Kirchhoff migration, did not result from a failure of the high frequency asymptotic approximation, but more from the simplicity of the approximations used in implementation. Many of them could be corrected by an improved numerical implementation or by introducing some extensions of high frequency approximations (as say in Gaussian beam migration).

On the other hand, WE migration also uses approximations introduced in order to reduce the computing time and to ensure the robustness of the simulation. Almost all of them are based on a one-way approximation and moreover use paraxial, phase shift or some other approximations. The favour for WE migration also comes from an easier numerical implementation, and it remains certainly a major advantage for the method. However, as asserted by Biondo, taking advantage of the non-asymptotic behaviour of WE migration requires estimation of the velocity macro-model without relying exclusively on high-frequency asymptotic approximations for either migration or velocity updating. For this purpose Biondo mentioned the Differential Semblance Optimisation (DSO) proposed by Bill Symes from Rice University. At that point Gilles retorted that the underlying philosophy in DSO also relied on an high frequency asymptotic assumption since it was still based on the concept of flat CIGs.

Following the first discussion period, the final four talks were presented.

Antonio Pica (CGG) presented "***3D multi-pathing and true amplitude Kirchhoff prestack depth migration***" (co-authored with Side Jin, Po Zhang and Peter Harris). Images of the SEG/EAGE 3D Salt Model compared favourably to images obtained by Bertrand Duquet (IFP) using 3D WE migration with the same model and dataset. The necessary smoothing of the velocity model (required by the wave front construction code) remained however a real problem, that had still to be tackled.

Yu Zhang (Veritas) next presented: "***True amplitude migration using common-shot one-way wavefield extrapolation***" (co-authored with James Sun, Samuel H. Gray, Carl Notfors, Norman Bleistein, & Guanquan Zhang). This addressed the adaptation to WE migration of true amplitude migration as developed by Norm Bleistein (Colorado School of Mines) in the context of Kirchhoff migration. Yu Zhang provided a beautiful illustration of the benefit that can be gained when comparing both approaches. The resulting formulae were given for a laterally invariant velocity model, but the extension to velocity macro-models with lateral variations was promised for the next SEG meeting in Salt Lake City.

Uwe Albertin (WesternGeco) then presented: "***Comparison between angle and offset gathers from wave equation migration and Kirchhoff migration***" (co-authored with Christof Stork, Phil Kitchenside, David Yingst, Clément Kostov, Brad Wilson, Dave Watts, Jerry Kapoor, & Gill Brown) discussing the various merits and limitations of Common Image Gathers obtained by WE migration and Kirchhoff migration. While CIG in offset remained easy to compute in Kirchhoff migration, in WE migration a convenient solution was given through CIGs in the angle domain. Both kinds of CIGs were compared on various dataset. These comparisons demonstrated the better quality of CIGs obtained by WE migration especially in complex media.

Finally Nanxun Dai (GXT) gave the last talk, entitled: "***An Adaptive phase shift and SSFPI method for pre stack depth migration***" (co-authored with Chris Willacy & Yong Sun). He presented a strategy for an accurate downward extrapolation in WE migration. An adaptative phase shift and Split-Step Fourier plus Interpolation algorithm was used with a careful adaptation to the complexity of the velocity model. The approach insured no numerical anisotropy, a low dispersion and a good image quality at wide angle. An illumination correction was also proposed. Comparative studies clearly demonstrated the better quality of the migrated images with respect to Kirchhoff migration or even with respect to other WE migration strategies, especially for steep dips.

After the final talk a general discussion concluded the interactive session, with all seven speakers joining the two chairmen. Rapidly the discussion evolved toward the analysis of the amplitude preservation in WE migration. Results presented by Yu Zhang (Veritas) were discussed and commented on in detail The audience, most noteably Norm Bleistein (Colorado School of Mines), Sam Gray (Veritas), Mihai Popovici (3DGeo) & Zvi Koren (Paradigm Geophysical) participated very actively in the discussion, promoting much debate.

Conclusions

By the end of the session, it was clear that research is still very actively on-going for both approaches. Although no definitive consensus was reached, a generalization that the co-chairmen would make is that both techniques can offer comparable image quality if sufficient effort is put into implementation. For integral techniques (e.g. Kirchhoff), this involves multi-arrival travel times calculations. WE techniques are algorithmically easier to get right, but are computationally more demanding. Model building remains a significant drawback for WE methods, unless various simplifying assumption are made (eg pre-conditioning the data with AMO, and then using a common azimuth WE migration).

Consequently, we infer that there is still much life left in the Kirchhoff technique even for environments where multi-pathing is an issue. And, that WE applications will gain more acceptance once adequate model building routes are available, especially for sub-salt plays.

Table 1 : Respective advantages and limitations of Integral methods and Differential methods.

Integral Methods	**Differential, Extrapolation or Continuation Methods**
Kirchhoff & Gaussian beam are the best known. Usually implemented in the time domain, but can be in the frequency domain. Distinguishing feature is separation of calculation of travel times from imaging Thus a subset of the image can be computed without needing to image the entire volume	Finite difference wavefield continuation is the best known, in conjunction with phase shift corrections. Each depth slice of the image is computed from the previously computed slice, thus the entire image volume needs to be formed. Dip response is dependent on the order of the expansion used (thus costly)
Strengths: - delivers sub-sets of the imaged volume, including offsets (*thus cost effective for iterative model building*) - good dip response	**Strengths:** - images all arrivals - simpler amplitude treatment
Weaknesses - Inherently kinematic - usually only delivers one arrival path - velocity field coarsely sampled for travel time computation, then arrival times interpolated back to seismic spacing	**Weaknesses** - images whole volume (thus costly) - obtaining good dip response is expensive - does not readily produce pre-stack data - thus difficult to achieve cost-effective iterative model building without ‘restrictive’ assumptions (eg mono-azimuth)